EFFICIENCY OF THROUGH-FLOW POROUS COOLING (BOUNDARY CONDITION OF THE THIRD KIND)

A. V. Kurpatenkov, V. M. Polyaev, and A. L. Sintsov

UDC 536.24

Expressions for calculating the coolant heating and the maximum wall temperature in a cooling channel filled with a porous material are given.

Heat exchange between the heated wall and the coolant is intensified considerably if the cooling channel is filled with a porous metal. This fact has been established theoretically, in particular, in [1], and experimentally, in [2].

In order to estimate theoretically the efficiency of such a cooling method, it is necessary to solve the well-known system of differential equations describing the temperature field within the porous insert in the channel. The solution of this problem has been treated in [1, 3-5]. Numerical methods requiring the use of computers and the development of a relatively complex program are described in [1, 3]. The investigations described in [4, 5] are devoted to an analytical solution of this problem. Two simplifying assumptions are used in [4]: 1) the thermal conductivity of the porous insert in the direction of coolant flow is zero; 2) the insert temperature is equal to the coolant temperature. These assumptions are used without any substantiation, which, in our opinion, somewhat diminishes the value of the results obtained. A similar problem has been solved in [5] on the basis of only one simplifying assumption (the first of the above two). As a result of using this assumption, the problem is reduced to a sequence of standard mathematical operations, whereby expressions for calculating the temperature fields in the form of sums of infinite series can be obtained. In our opinion, there are two obstacles to the utilization of the results obtained in [5]: first, a lack of justification for using the above assumption and, second, the necessity of summing a rather complex infinite series.

We propose elementary equations for calculating the coolant heating and the maximum wall temperature of a cooling channel filled with a porous metal on the basis of the numerical solution of this problem [3] and its analysis.

We shall now describe the calculation procedure. We contemplate a cooling channel consisting of the annular region between two coaxial tubes. It is assumed that the temperature field in the channel is described by the system of equations

$$\varkappa\partial^2T/\partial x^2+\partial^2T/\partial y^2+y^{-1}\partial T/\partial y=\sigma\pi(T-\tau),\quad \partial\tau/\partial x=\sigma(T-\tau). \tag{1}$$

The boundary condition at the inside wall of the channel, whose dimensionless radius is $y_n$, is the following:

$$-\partial T_n/\partial y=\beta_n(1-T_n). \tag{2}$$

The outside wall of the channel is assumed to be thermally insulated: $\partial T_V/\partial y = 0$.

The condition at the channel inlet is given by

$$-\partial T_a/\partial x=\sigma_a\pi_x(\tau_- - T_a). \tag{3}$$

We have a similar condition at the surface of the channel outlet:

$$-\partial T_b/\partial x=\sigma_b\pi_x(T_b-\tau_b).$$

On the basis of the elementary energy balance, the coolant temperature at the surface of the channel inlet and the temperature at a point remote from the surface of the channel outlet are given by

$$\tau_a=\sigma_aT_a+(1-\sigma_a)\tau_-,\quad \tau_+=\sigma_bT_b+(1-\sigma_b)\tau_b. \tag{4}$$

N. E. Bauman Moscow Technical College. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 52, No. 9, pp. 361-365, September, 1987. Original article submitted July 25, 1986.

TABLE 1. Approximation Coefficients $A_i$ (odd columns) and $a_i$ (even columns)

| $y_n=0,1$ | | $y_n=0,5$ | | $y_n=2,0$ | | $y_n=\infty$ | |
|---|---|---|---|---|---|---|---|
| | | | $\beta_n=0,01$ | | | | |
| —2,42 | —2,78 | —2,05 | —2,30 | —1,90 | —2,10 | —1,83 | —2,00 |
| —0,549 | —0,978 | —0,639 | —0,971 | —0,681 | —0,962 | —0,700 | —0,956 |
| 0,133 | 7,15—3 | 0,103 | —5,18—3 | 8,52—2 | —1,31—2 | 7,57—2 | —1,79—2 |
| | | | $\beta_n=0,1$ | | | | |
| —1,42 | —1,78 | —1,06 | —1,30 | —0,918 | —1,11 | —0,856 | —1,02 |
| —0,499 | —0,929 | —0,531 | —0,865 | —0,542 | —0,825 | —0,546 | —0,804 |
| 9,89—2 | —2,63—2 | 3,42—2 | —7,34—2 | —2,00—3 | —9,64—2 | —1,35—2 | —0,107 |
| | | | $\beta_n=1,0$ | | | | |
| —0,525 | —0,867 | —0,299 | —0,522 | —0,229 | —0,395 | —0,200 | —0,338 |
| —0,315 | —0,749 | —0,280 | —0,615 | —0,265 | —0,549 | —0,259 | —0,516 |
| 9,50—2 | —0,118 | —4,50—2 | —0,158 | —6,53—2 | —0,170 | —7,41—2 | —0,175 |
| | | | $\beta_n=10$ | | | | |
| —8,74—2 | —0,387 | —3,19—2 | —0,212 | —1,81—2 | —0,144 | —1,30—2 | —0,113 |
| —8,72—2 | —0,515 | —6,18—2 | —0,385 | —5,51—2 | —0,324 | —5,25—2 | —0,295 |
| —1,59—2 | —0,152 | —2,49—2 | —0,148 | —2,69—2 | —0,141 | —2,77—2 | —0,137 |
| | | | $\beta_n=100$ | | | | |
| —9,05—3 | —2,286 | —2,83—3 | —0,167 | —1,43—3 | —0,115 | —9,26—4 | —8,98— |
| —1,07—2 | —0,427 | —6,80—3 | —0,318 | —5,84—3 | —0,264 | —5,49—3 | —0,237$^{2}$ |
| —2,71—3 | —0,141 | —3,05—3 | —0,126 | —3,09—3 | —0,116 | —3,09—3 | —0,110 |

Remark. For brevity, the decade base number has been omitted, and only the exponent is given. Thus, for instance, 9.50 — 3 = $9.50 \cdot 10^{-3}$.

In particular, it follows from (4) that $\sigma_a$ and $\sigma_b$ vary in the range from zero to unity.

The values of $T_{max}$ and $\Delta\tau$ are of the greatest interest in calculating through-flow porous cooling. For brevity, we shall refer to these values as the process characteristics. Analyzing the above expressions, we arrive at the conclusion that the process characteristics are generally functions of the following arguments: $\varkappa$, $\pi$, $\sigma$, $y_n$, $\sigma_a$, and $\sigma_b$. There are evidently too many of them for establishing sufficiently simple correlations. We shall try to reduce their number.

First of all, we shall limit the $\varkappa$ number to the following range: $0 \leqslant \varkappa \leqslant 1$. In this case, calculations show that the variation of $\varkappa$, with the other numbers held constant, hardly alters the values of the process characteristics. This makes it possible to eliminate $\varkappa$ as an argument. Furthermore, calculations show that variation of the $\sigma_a$ and $\sigma_b$ values from zero to unity also exerts a negligible effect on the process characteristics. Thus, $T_{max}$ and $\Delta\tau$ are essentially functions of three arguments, $\pi$, $\sigma$, and $y_n$. There is also the possibility of eliminating $\sigma$ from the number of arguments: calculations indicate that, beginning with a $\sigma$ value approximately equal to 10, the process characteristics, for practical purposes, no longer depend on this dimensionless number. Thus, in order to eliminate $\sigma$ it is sufficient to assume that $\sigma \geqslant 10$.

On the basis of the above, we reach the conclusion that the process characteristics can be estimated with sufficient accuracy with respect to the assigned $\pi$ and $y_n$ values. Figure 1 shows the values of $T_{max}$ and $\Delta\tau$ as functions of $\pi$ for the particular case $y_n = \infty$.

For any $y_n$, the dependence of the process characteristics on $\pi$ is approximated in the following manner:

$$F = A_1 + A_2 z + A_3 z^2, \quad f = a_1 + a_2 z + a_3 z^2. \tag{5}$$

The values of the approximation coefficients $A_i$ and $a_i$ are given in Table 1. As an example, we shall determine the specific form of (5) for $y_n = 0.5$ and $\beta_n = 0.1$. As a result, we obtain

$$F = -1{,}06 - 0{,}531z + 0{,}0342z^2, \quad f = -1{,}30 - 0{,}865z + 0{,}0734z^2.$$

It should be noted that the diagrams in Fig. 1 and the approximations provided hold for $\sigma \geqslant 10$ and $0 \leqslant \varkappa \leqslant 1$, regardless of the values of $\sigma_a$ and $\sigma_b$. The value of $\pi$ varies from 0.01 to 100.

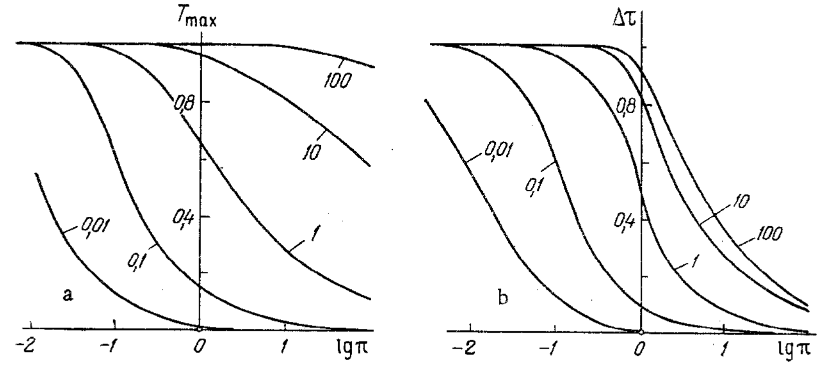


Fig. 1. Dependence of $T_{max}$ (a) and $\Delta\tau$ (b) on $\pi$ and $\beta_n$ for $y_n = \infty$: $0 \leqslant \varkappa$, $\sigma_a$, $\sigma_b \leqslant 1$; $\sigma \geqslant 10$. The figures at the curves represent the $\beta_n$ values.

## NOTATION

T and $\tau$, dimensionless temperatures of the insert material and the coolant, respectively: $T = (T' - \tau'_-)/T$, $\tau = (\tau' - \tau'_-)/T$, $T = \Theta' - \tau'_-$; x, dimensionless coordinate directed along the coolant flow: $x = x'/L$; L, length of the porous insert; y, dimensionless coordinate perpendicular to the x axis: $y = y'/H$; H, height of the porous insert' $F = \log T_{max}$, $f = \log \Delta\tau$; $\Delta\tau = \langle\tau_+\rangle - \tau_-$, $z = \log \pi$; $A_i$, $a_i$, coefficients; $\lambda_x$ and $\lambda_y$, thermal conductivity coefficients of the porous material in the corresponding directions; $\alpha_v$, volumetric heat-transfer coefficient in the porous material; $\dot{m}$, ratio of the coolant discharge to the cross-sectional area of the channel; $c_p$, specific heat of the coolant; $\alpha_n$, heat-transfer coefficient characterizing heat exchange between the inner wall of the channel and a certain medium; $\alpha_{a(b)}$, heat transfer coefficient characterizing heat exchange between the coolant and the inlet (outlet) surface of the insert. Dimensionless numbers: $\varkappa = \lambda_x H^2(\lambda_y L^2)^{-1}$, $\sigma = \alpha_V L(\dot{m}c_p)^{-1}$, $\pi = \dot{m}c_p H^2(\lambda_y L)^{-1}$, $\beta_n = \alpha_n H/\lambda_y$, $\pi_x = \dot{m}c_p L/\lambda_x$, $\sigma_{a(b)} = \sigma_{a(b)}/(\dot{m}c_p)$. Superscripts or subscripts: the prime (') denotes a dimensional quantity; n or v, quantities pertaining to the inner or outer wall of the channel; *a* or b, quantities pertaining to the inlet or outlet surface of the porous insert; − or +, quantities pertaining to a region remote from the inlet or outlet of the porous insert; max, maximum; brackets: $\langle\ldots\rangle$, quantity averaged over the insert height; $\Theta'$, ambient temperature.

## LITERATURE CITED


1. J. Koch and R. Coloni, Teploperadacha, 96, No. 3, 66-74 (1974).
2. J. Koch and R. Stevens, Teploperedacha, 97, No. 2, 153-154 (1975).
3. A. V. Kurpatenkov, V. M. Polyaev, and A. L. Sintsov, Inzh.-Fiz. Zh., 47, No. 6, 984-991 (1984).
4. V. A. Maiorov, V. M. Polyaev, L. L. Vasil'ev, and A. I. Kiselev, Inzh.-Fiz. Zh., 47, No. 1, 13-24 (1984).
5. V. A. Maiorov, V. M. Polyaev, and L. L. Vasil'ev, Inzh.-Fiz. Zh., 47, No. 2, 199-204 (1984).